Fairness in Kidney Exchange Programs: the Nash Social Welfare Program Perspective

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Motivation

1 in 10 Canadians have a chronic kidney disease. From 2008 to 2018, there was a 35% increase in the number of Canadians (excluding Quebec) living with end-stage kidney disease.

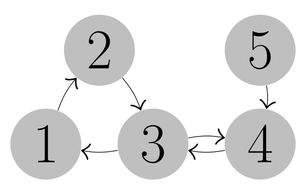
$50 billion per year is the approximate cost to the Canadian health care system for chronic kidney disease. Dialysis alone corresponds to significant expenditures in the countries health systems.

4 years is the median waiting time for a deceased-donor kidney transplantation (shortest Nova Scotia: 3 years; longest Manitoba: 5.7 years).

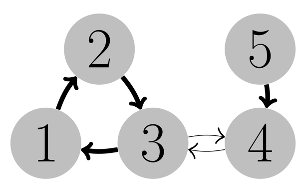
Kidney Exchange Programs (KEPs) represent an additional possibility for patients waiting for a transplant and who have an incompatible donor (friend or relative).

Kidney Exchange Problem

A KEP instance can be represented as a graph $G = (V, A)$, where V is the union of incompatible pairs P and altruistic donors N, and A is the set of arcs representing compatibilities.



The goal of KEPs is to determine a set of disjoint cycles and chains (paths starting in an altruistic donor) representing the kidney exchanges to be performed (Abraham et al., 2007; Dickerson et al., 2016; Roth et al., 2007).



Utility and Fairness

- Often the objective is to maximize the number of transplants. Multiple optimal exchanges can exist.
- This does not take into account any fairness notion in the solution. We are just maximizing utility.
- We can define multiple concepts of fairness in KEPs, which we will refer to as fairness schemes.

Fairness Schemes

It has been demonstrated in Bolton et al., 2005 that individuals are indifferent about fair outcomes and fair procedures. This justifies our use of distributions over feasible exchange plans. We use the notation $\mathcal{L}(U)$ to refer to the vector of utilities to each pair $v \in P$ when following the fairness scheme $\mathcal{L}$ and the set of possible utilities is U.

With this in mind, we can define various fairness schemes in KEPs:

- **Rawls** (Rawls, 1973): We aim to maximize the mininum probability of being selected in an exchange plan among pairs $v \in P$.
- **Nash** (Nash, 1950): An exchange plan is fair according to Nash, if it satisfies the following condition:
$$\sum_{v \in P} \frac{u_v - \mathcal{L}(U)_v}{\mathcal{L}(U)_v} \leq 0 \quad \forall u \in U$$
We can define a fairness score by finding the value of the left-hand side. It will give us a measure of how much we violate this constraint.
- **L_p fairness** (Farnadi et al., 2021): With this measure, we seek to minimize the distance from the average probability of being selected as part of an exchange plan among pairs $v \in P$.

These fairness schemes each have their pros and cons. Usually, the main takeaway is that optimizing a KEP only with respect to a chosen fairness scheme can lead to a massive loss of utility. How can we combine utility and fairness schemes in balanced approach?

Methodology

We combine utility and fairness using the *Nash Social Welfare Program* (NSWP) Charkhgarda et al., 2020. It has the general form:

$$\max \prod_{i=1}^{k} (f_i(x) - d_i)$$
$$\text{s.t.} \quad x \in X$$
$$f_i(x) \geq d_i \forall i = 1, \dots, k$$

where X is the set of constraints and the vector d is a reference point chosen beforehand.

Column generation: Since we are searching for a distribution over feasible exchange plans, we can use a column generation approach to enumerate them. The NSWP, when adapted to a KEP instance, can take the form of an SOCP with the Rawlsian or Nash fairness schemes.

Analysis of the solution: We can measure how well the solution performs in terms of both objectives. We use a measure called the *Price of fairness* (POF) Bertsimas et al., 2011:

$$\frac{\sum_{v \in P} u_v^* - \sum_{v \in P} \mathcal{L}(U)_v}{\sum_{v \in P} u_v^*}$$

where $u^* \in U$ gives the maximum value of $\sum_{v \in P} u_v^*$. We also define a similar measure that captures the reverse cost, i.e. the *Price of utility* (POU).

Results

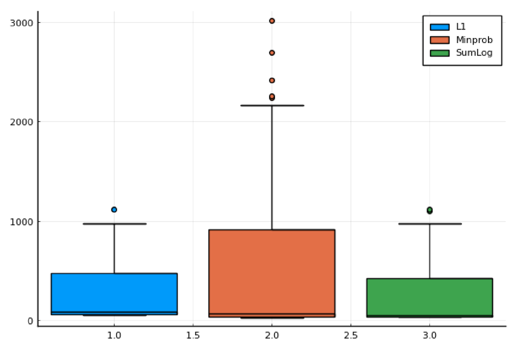
We evaluate our fairness measures on a dataset consisting of KEPs with 16 to 256 vertices in their graphs. For brevity, we present results over graphs of size 64. We investigate various research questions:

Q. How does the NSWP perform in terms of the POF and the POU? How many solutions are in the support?

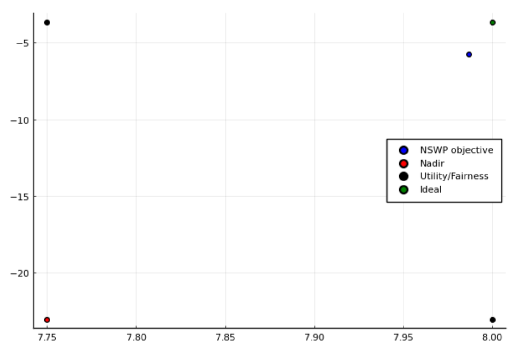
Scheme	POF	POU	Solutions
Nash	0.233%	26.107%	181.23
Rawls	5.494%	21.39%	23.13

Averages over graphs with $|P| = 64$

Q. How efficient is the method when applied to various instances in terms of runtime?



Q. Where is the solution on the Pareto front?



Conclusions & Future Directions

- The NSWP provides a framework to combine multiple objectives. When applied to KEPs, we are able to balance utility and some fairness scheme in a rigorous manner.
- In future research, we might want to extend this formulation to the dynamic setting, i.e. varying through time.

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